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THEORY OF AN ANISOTROPIC THERMOELECTRIC REFRIGERATOR IN A MAGNETIC FIELD

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It is shown that the difference between the specific thermal conductivity coefficients in the absence of an electric current and field has a substantial effect on the maximum temperature drop.

The authors of [1, 2], who considered problems of the theory of an anisotropic thermoelectric refrigerator in a magnetic field, confine themselves to the assumption that the specific thermal conductivity coefficients in the absence of an electric current $\hat{\kappa}_j(\mathrm{H})$ and in the absence of an electric field $\hat{\kappa}_\mathrm{E}(\mathrm{H})$ are equal. This means that the authors of these works neglect the second term in the relation [3]

$$\hat{\kappa}_j(\mathrm{H}) = \hat{\kappa}_\mathrm{E}(\mathrm{H}) - \hat{\Pi}(\mathrm{H})\,\hat{\sigma}(\mathrm{H})\,\hat{\alpha}(\mathrm{H})\,. \tag{1}$$

On the one hand, this approximation is only valid for thermoelectric materials (TM) with a low thermoelectric Q-factor, and, on the other hand, for highly efficient TM it ignores the temperature dependence of the coefficients $\hat{\kappa}_j(\mathrm{H})$ contained in Thomson's second relation:

$$\Pi_{ik}(\mathrm{H}) = T\alpha_{ki}(-\mathrm{H})\,. \tag{2}$$

As will be shown later, systematic account for the indicated term from (1) in the theory, leads to essentially new results, in particular, to a different value of the maximum temperature drop ΔT.

In the present work we dwell in detail on estimation of ΔT, considering the same model of an anisotropic thermoelement (ATE) as in [1].

The heat conduction equation that systematically takes into account (1) has the following form for Samoilovich and Slipchenko's ATE model [1]:

$$(1 - Z_\mathrm{E}^* T)\,\frac{d^2T}{dy^2} - Z_\mathrm{E}^*\left(\frac{dT}{dy}\right)^2 - KZ_\mathrm{Ea}(1+\delta)\,\frac{dT}{dy} + K^2\delta Z_\mathrm{Ea} = 0\,. \tag{3}$$

Here

$$K = \frac{E}{\alpha_{12}(H)}\,;\quad \delta = \frac{\alpha_{12}(H)}{\alpha_{12}(-H)}\,; \tag{4}$$

$$Z_\mathrm{E}^* = Z_\mathrm{E} f_\mathrm{b} - Z_\mathrm{Ea}\,;\quad Z_\mathrm{E}(H) = \frac{\sigma_{11}(H)\,\alpha_{11}^2(H)}{\kappa_{11}^\mathrm{E}(H)}\,; \tag{5}$$

$$Z_\mathrm{Ea}(H) = \frac{\alpha_{12}(-H)\,\alpha_{12}(H)}{\kappa_{11}^\mathrm{E}(H)\,\rho_{11}(H)}\,;$$

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$$f_b(H) = 1 + \delta(H)\left(\frac{\alpha_{12}(-H)}{\alpha_{11}(H)}\right)^2 + \frac{\alpha_{12}(-H)}{\alpha_{11}(H)}\frac{\sigma_{12}(H)}{\sigma_{11}(H)} + \frac{\alpha_{12}(H)}{\alpha_{11}(H)}\frac{\sigma_{12}(-H)}{\sigma_{11}(H)}. \tag{6}$$

It should be noted that unlike [1, 2], the thermoelectric Q-factor is determined here in terms of $\kappa_{11}^{E}(H)$. According to the second law of thermodynamics, this implies that

$$Z_E(H) f_b T < 1; \quad |Z_{Ea}(H) T| < 1.$$

The following expression is a general integral of Eq. (3):

$$\frac{A-1}{A}\ln\left(\frac{1}{2}(1+\delta) Z_{Ea} T\left(1 + A + \frac{1-A}{Z_E^* T}\right) - \delta Z_{Ea} Ky + C_1\right) + \\ + \frac{A+1}{A}\ln\left(\frac{1}{2}(1+\delta) Z_{Ea} T\left(1 - A + \frac{1+A}{Z_E^* T}\right) - \delta Z_{Ea} Ky + C_1\right) = 0, \tag{7}$$

where

$$A = \sqrt{\left(1 + \frac{4\delta}{(1+\delta)^2}\frac{Z_E^*}{Z_{Ea}}\right)}. \tag{8}$$

Using expression (7) and the boundary conditions from [1] we obtain for the maximum temperature drop ΔT the following:

$$\Delta T = T_0 \frac{1 - Z_E^* T_0 - (1 - Z_E^* T_0 \tau^-)^{\frac{A-1}{2A}}(1 - Z_E^* T_0 \tau^+)^{\frac{A+1}{2A}}}{Z_E^* T_0}, \tag{9}$$

where

$$\tau^- = 1 + \frac{2}{(1+\delta)(A-1)}; \quad \tau^+ = 1 - \frac{2}{(1+\delta)(A+1)}. \tag{10}$$

Let us now consider some particular cases.

I. Isotropic medium ($Z_{Ea} = 0$). In this case $A \to \infty$ and therefore from Eq. (9) it follows that $\Delta T \to 0$.

II. Gyrotropic medium ($Z_{Ea} = -Z_{EQ}$). For this medium

$$\delta = -1, \quad \frac{A-1}{A} = \frac{A+1}{A} = 1,$$

$$\tau^- = 1 + \sqrt{\left(\frac{Z_{EQ}}{Z_E^*}\right)}, \quad \tau^+ = 1 - \sqrt{\left(\frac{Z_{EQ}}{Z_E^*}\right)},$$

and therefore from Eq. (9) we have

$$\Delta T = \frac{1}{2}\frac{\bar{Z}_{EQ} T_0^2}{1 - \bar{Z}_{EQ}\bar{T}}. \tag{11}$$

Here

$$\bar{Z}_{EQ} = \frac{Z_{EQ}}{1 - Z_E f_b \bar{T}}; \quad Z_{EQ} = \frac{\alpha_{12}^2(H)}{\kappa_E(H)\rho(H)}.$$

TABLE 1. Values for the Maximum Temperature Drop ΔT

$Z_{Ea}T_0$	$Z_E f_b T_0$					
	0	0.1	0.2	0.3	0.4	0.5
0	0	0	0	0	0	0
0.003	0.150	0.1668	0.1876	0.2144	0.2502	0.3003
0.02	1.506	1.674	1.884	2.156	2.521	3.035
0.05	7.627	8.504	9.612	11.06	13.04	15.94
0.09	13.91	15.56	17.65	20.44	24.33	30.23
0.10	15.51	17.36	19.72	22.86	27.28	34.05
0.15	23.67	26.58	30.36	35.48	42.90	54.92
0.20	32.10	36.19	41.56	48.98	60.11	79.40
0.25	40.83	46.21	53.37	63.48	79.20	108.8
0.30	49.86	56.65	65.83	79.07	100.5	145.2
0.35	59.20	67.55	78.99	95.88	124.5	192.7
0.40	68.87	78.92	92.89	114.0	151.8	260.1
0.45	78.87	90.79	107.6	133.7	183.1	374.8
0.50	84.23	103.19	123.2	155.2	219.6	∞

In the conventional theories [4] $Z_E f_b T \ll 1$ and consequently $\overline{Z}_{EQ}T = Z_{EQ}T \ll 1$, $\Delta T = \frac{1}{2} Z_{EQ} T_1^2$.

III. Anisotropic medium ($Z_E f_b \to 0$; $A = 1 - \delta/1 + \delta$). The case under consideration corresponds to the approximation of [1]. Having written formula (9) for the conditions in question, we obtain

$$\Delta T = T_0 \frac{(1 + Z_{Ea} T_0 \Theta)^{\frac{1}{\Theta}} - 1 - Z_{Ea} T_0}{Z_{Ea} T_0}, \quad \Theta = \frac{\delta - 1}{\delta}. \tag{12}$$

It is easy to see that formula (12) is fully equivalent to the formula

$$\Delta T = T_1 \left[1 - \frac{(1 + Z_{Ea} T_1)^{\Theta} - 1}{\Theta Z_{Ea} T_1} \right]$$

from [1], if $Z_{Ea} \equiv Z$ is assumed.

In the case where the magnetic field is absent ($\delta \to 1$, $\Theta \to 0$) we find from (12) in the limit

$$\Delta T = T_0 \frac{e^{Z_{Ea} T_0} - 1 - Z_{Ea} T_0}{Z_{Ea} T_0}, \tag{13}$$

which is also equivalent to the formula

$$\Delta T = T_1 - \frac{1}{Z} \ln (1 + Z T_1)$$

from the same work.

IV. Anisotropic medium ($H = 0$). Here the expression for ΔT coincides with (9) with the following values of the parameters:

$$\delta = 1, \quad A = \sqrt{\left(\frac{Z_E f_b}{Z_{Ea}} \right)}, \quad \tau^- = \frac{A}{A - 1}, \quad \tau^+ = \frac{A}{A + 1}.$$

From Eq. (9) it follows that indeterminacy arises at $Z_E^* T_0 = 0$. For the case in question this occurs when $Z_E f_b T_0 = Z_{Ea} T_0$. It is easy to verify that under this condition

$$\Delta T = -\frac{T_0}{2}\left[1 + \frac{\ln(1 - 2Z_{Ea}T_0)}{2Z_{Ea}T_0}\right], \tag{14}$$

from which we find that $Z_{Ea}T_0 < 1/2$.

Table 1 contains the quantity ΔT for particular values of the parameters $Z_E f_b T_0$ and $Z_{Ea}T_0$. The results given in Table 1 show that the value of the maximum temperature drop ΔT differs substantially from the values of ΔT obtained from the theory of [1], especially in the range of large values of $Z_{Ea}T_0$ and $Z_E f_b T_0$.

Thus, the present theory of a solid-state anisotropic refrigerator indicates that the expressions for the energy characteristics of thermoelectric transforming devices coincide with results obtained earlier by various authors only in the limiting case of equal specific thermal conductivity coefficients $\hat{\kappa}_j$ and $\hat{\kappa}_E$. In view of the prospects for thermoelectric power conversion, caused, in particular, by improvements in the production processes for new highly efficient TM, it can be concluded from the data reported in the present work that concrete calculations should include the difference between the coefficients in question.

NOTATION

$\hat{\kappa}_j(\mathbf{H})$, $\hat{\kappa}_E(\mathbf{H})$, specific thermal conductivity coefficients; **H**, magnetic field intensity vector; $\hat{P}(\mathbf{H})$, Peltier tensor; $\hat{\delta}(\mathbf{H})$, specific electric conductivity tensor; $\hat{\alpha}(\mathbf{H})$, thermoelectromotive force tensor; ΔT, maximum temperature drop; T, temperature; y, coordinate; K, δ, anisotropy parameters; Z, thermoelectric Q-factor; $f_b(\mathbf{H})$, function defining anisotropic media; C_1, integration constant; T_0, T_1, temperatures of the cold and hot faces of the ATE. Subscripts: i, k, tensors; j, E, density and strength of the electric field.

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